D2

APPENDIX A

A Small Free-Piston Stirling Refrigerator

by

A. K. de Jonge

(14th Intersociety Energy Conversion Engineering Conference,
Boston, Mass, Aug. 5-10, 1979)

A SMALL FREE-PISTON STIRLING REFRIGERATOR

A.K. de Jonge
Philips Research Laboratories
Eindhoven, The Netherlands

ABSTRACT

Theoretical studies have been done on the performance characteristics of Stirling machines with free moving pistons. This has amongst others resulted in the development of a small Stirling refrigerator with a capacity of 1 W at 77K driven by a linear electric motor. The unit is a hermetically closed machine with helium as working medium, having no bearings or crankshafts and no side forces on the piston and displacer.

These design features have resulted in a long lifetime. After a description of the design of the small free piston Stirling refrigerator, a new way to describe the pressure fluctuations in the Stirling system is given. Using this description and the characteristics of the linear motor the equations of movement of the piston and the displacer are derived. After this, experimental results are given.

WITHIN THE PHILIPS CONCERN a small cryogenic cooler, based on the free displacer Stirling principle, has been designed and developed with a capacity of 1 W at 77K. The main application is in the field of infrared radiation detection. In the design of the machine with a free piston, a free displacer and a linear motor only two sliding seals on piston and displacer are present which need no lubrication. With the linear motor inside the crankcase the machine can be hermetically sealed. The whole concept leads to a machine with a very long life requiring no maintenance and having negligible degradation.

The machine, which is produced under the name MC.80 [1*], has indeed shown a long lifetime and a very stable lowest temperature (under the name MMC-80 a military version of the machine is also available from Philips USFA, Eindhoven).

This paper gives a simplified description of the pressure fluctuation in the machine which leads to an explanation of the behaviour of a free displacer refrigerator. Also the linear drive system of the machine will be discussed.

DESCRIPTION OF THE CRYOGENIC COOLER

A schematic drawing of the machine is given in fig. 1. The machine consists of a piston unit and a cold finger part. The piston is directly coupled to the moving coil of the linear motor. This linear motor consists of a permanent magnet system (ferroxdure is used in the MC.80, samarium-cobalt in the MMC-80). A roughly constant field is made in the gap around the iron inner cylinder. In this gap the coil driving the piston moves up and down as the result of an AC current through the coil. The centre position of the coil (and piston) is held by a small mechanical spring. All forces on the piston are in the direction of the movement, so no side forces are present and a dry running material can be used as seal.
The cold finger of the machine consists of a displacer with a diameter of 9.4 mm (in the MMC-80) moving up and down in the cylinder. The displacer contains an inside gauze regenerator whereas the gaps between displacer and outer cylinder at the bottom part and the upper part are acting as cooler and freezer respectively. The displacer is

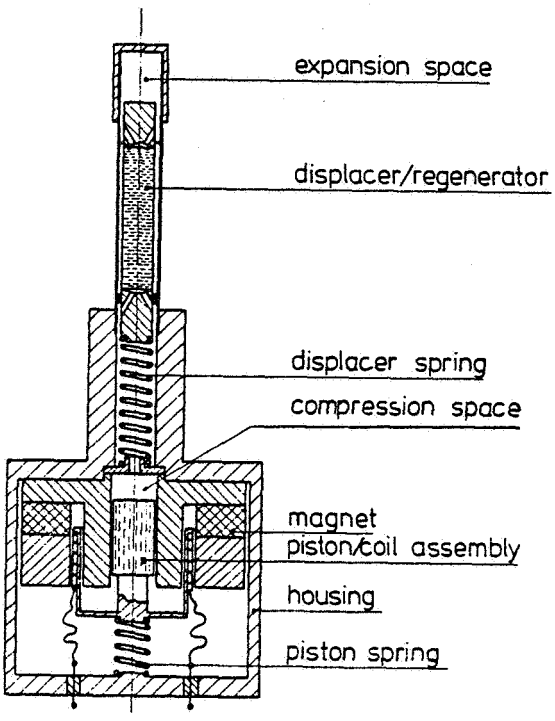

Fig. 1. - Schematic cross section of the MC.80 Stirling refrigerator with electrodynamic drive.

*) Numbers in [ ] designate References at end of paper.

8412-0513-2/79/0779-245 $01.00/0
Copyright 1979 American Chemical Society

mounted on a mechanical spring.
Here again only forces in the direction of movement are present and so the displacer seal can be dry running too.

THE PRESSURE FLUCTUATION IN A STIRLING SYSTEM

In previous publications about the Stirling theory (see e.g. [2] and [3]) the total pressure variation in the engine is given as a function of the movements of the two moving elements, the phase between the movements and the temperatures and sizes of all spaces in the engine. As in this engine the strokes nor the phases are known from the start, it is better to calculate the fluctuation as a vectorial sum of different fluctuations due to the different movements. The four main causes of pressure variations are:

1. The piston position determines the total volume.
2. The displacer position determines the amount of gas in the expansion and compression space, so influences the pressure.
3. The piston movement gives rise to gasflow causing pressure differences in the spaces.
4. The displacer movement again leads to flow with the same effects as under 3.

Ad 1. Keeping the displacer in a fixed position and moving the piston leads to a pressure [2], [3]

$$p = p_m + \frac{p_m S_p}{V_o B} y = p_m + C_y y \qquad (1)$$

in which
p_m = average cycle pressure
S_p = piston surface area
y_p = piston position
$V_o B$ = total cycle volume, normalised to T_c

$$B = \sum_i \frac{V_i \tau_i}{V_o}, \quad \tau_i = \frac{T_c}{T_i}$$

T_c = temperature compression space

Ad 2. If the piston is in a constant position and the displacer moves, the pressure will be given by:

$$p = p_m + \frac{p_m S_d (\tau - 1) x}{V_o B} = p_m + C_x x \qquad (2)$$

in which S_d is the displacer cross-sectional area, $\tau = T_c/T_e$ the temperature ratio between the two varying spaces and x the displacer position. Change of the position of the displacer gives rise to a pressure change as gas from a space of one temperature, e.g. T_c, is moved to a space of temperature T_e.

The cycle pressure, not disturbed by flow loss, can be given by

$$p = p_m + C_y y + C_x x \quad [4] \qquad (3)$$

Ad 3 and 4.
The flow losses give rise to different pressures in different parts of the engine. The pressure difference across a certain resistance due to the flow can be given by (assuming laminar flow)

$$\Delta p = \frac{c_1 \eta}{\rho v d_h} \frac{l}{d_h} \frac{1}{2} \rho v^2 = c.v$$

in which
l = length of resistance
d_h = characteristic diameter
ρ = gas density
v = gas velocity
η = viscosity
c and c_1 are constants.
The velocity of the gas at some point in the engine is the vectorial sum of two velocities proportional to the velocity of the piston $\dot{y}$ and that of the displacer $\dot{x}$ respectively.
The pressure difference across a certain flow resistance can be translated into pressure changes on both sides of the resistance.

All this leads to the total pressure fluctuation in the variable compression and expansion volumes of:

$$p_c = p_m + C_y y + C_x x + C_{pc} \dot{y} + C_{dc} \dot{x}, \qquad (4)$$

$$p_e = p_m + C_y y + C_x x + C_{pe} \dot{y} + C_{de} \dot{x} \qquad (5)$$

and $p_e - p_c = (C_{pe} - C_{pc})\dot{y} + (C_{de} - C_{dc})\dot{x} = C_{pp}\dot{y} + C_{dd}\dot{x}$

The simple description of the pressures given here is only valid if the constants C_y, C_x, C_{dd} and C_{pp} are independent of x, y, $\dot{x}$ and $\dot{y}$, which is not quite true but is a reasonable approximation as shown below. There are also other causes of pressure variations such as piston leakage, temperature variations due to heat transfer to walls and bad mixing, change of average temperatures in heat exchangers and varying spaces due to load changes etc. Also the flow is not laminar everywhere so flow resistances are not linearly dependent on gas velocities.

A numerical investigation has been made based on our existing Stirling computer cal-

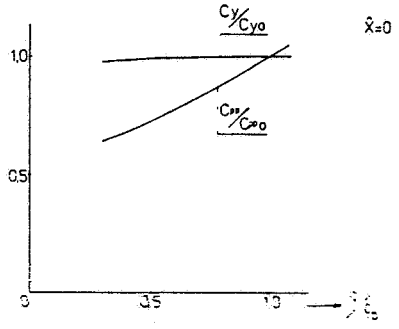

Fig. 2. - Relative variation of C_y and C_{pp} versus relative piston amplitude

culations on the MMC-80 how the "constants" mentioned above vary. The variations of C_y C_{pp} as a function of the amplitude of the piston $\hat{y}$ (displacer in a fixed position) are plotted in fig. 2. C_y varies slightly due to a change in temperature at different places in the engine. C_{pp} varies more due mainly to the non-laminar $_{pp}$flow.
The variations of C_x and C_{dd} are plotted in fig. 3 as a function of the amplitude of the displacer $\hat{x}$ (piston in a fixed position). This again indicates only a slight variation in C_x and more in C_{dd}.

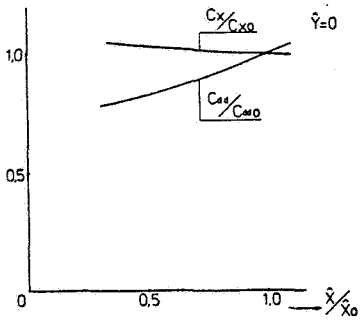

Fig. 3. - Relative variation of C_x and C_{dd} as a function of relative displacer amplitude

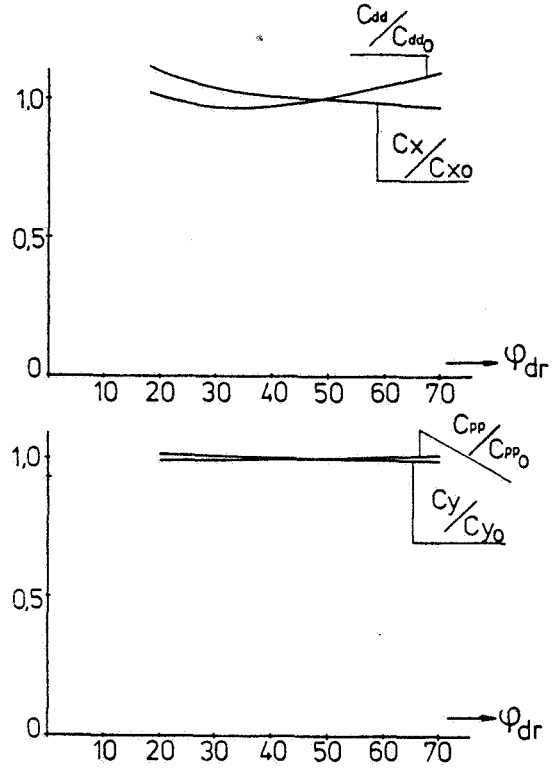

Fig. 4. - Effect of phase shift on the constants C_x, C_{dd}, C_y and C_{pp} with constant piston and displacer amplitude

In figure 4 the variations of the constants are calculated as a function of the phase angle (φ_{dr}) between displacer and piston movement. The variations are small.

CALCULATION OF PRODUCTION AND INPUT POWER

With the above mentioned expression for the pressure it is easy to calculate the ideal cold production (Q_{eo}) and ideal input power (N_p) of a machine.

$$Q_{eo} = n \oint p \, dV_d = - n\pi C_y \hat{x} \hat{y} S_d \sin \varphi_{dr} \qquad (6)$$

where
$dV_d = S_d dx$ = volume variation of expansion volume
n = number of cycles per second
$x = \hat{x} \cos \alpha$
$y = \hat{y} \cos (\alpha - \varphi_{dr})$
$\alpha = \omega t$
ω = angular frequency

Only the pressure fluctuation due to the piston movement (C_y) is important for Q_{eo}.

The input power is given by

$$N_p = n \oint p dV_p = n \pi C_x \hat{x} \hat{y} S_p \sin \varphi_{dr}, \qquad (7)$$

with $dV_p = S_p dy$,

indicating that only C_x, the pressure fluctuation due to the displacer movement, gives N_p. If $C_x > 0$, which is the case if $\tau > 1$, N_p is positive when $\sin \varphi_{dr}$ is also $> o$, which is the case for a refrigerator.
If $\tau < 1$, then the machine is a hot gas engine. Then $C_x < 0$ and N_p is negative with positive $\sin \varphi_{dr}$.

MOVEMENT OF THE FREE DISPLACER

The equation of movement of the displacer is given by

$$M_d \ddot{x} + (p_e - p_c) S_d + C_{vd} x = 0$$

in which $M_d\ddot{x}$ is the acceleration force (M_d is the displacer mass), $(p_e - p_c)S_d$ is the driving force on the displacer and $C_{vd}x$ is the spring force (friction is neglected).

With the pressure equations this leads to

$$M_d\ddot{x} + C_{dd}S_d\dot{x} + C_{vd}x = (-C_{pp})S_d\dot{y}, \quad (C_{pp} < 0).$$

Now this is the equation for a damped oscillator with a driving force. The driving force $(-C_{pp})S_d\dot{y}$ is due to the piston movement, the damping force $C_{dd}S_d\dot{x}$ to the displacer movement.
With $y = \hat{y} \cos (\alpha - \varphi_{dr})$ and $x = \hat{x} \cos \alpha$:

$$M_d(\omega^2 - \omega_d^2)\hat{x} \cos \alpha + C_{dd}S_d\hat{x}\,\omega \sin \alpha =$$
$$(-C_{pp})S_d\omega\hat{y} \sin (\alpha - \varphi_{dr})$$

with $\omega_d^2 = \frac{C_{vd}}{M_d}$

Then $\tan \varphi_{dr} = \frac{M_d(\omega_d^2 - \omega^2)}{C_{dd}S_d\,\omega}$ (8)

and $\hat{x} = C_{pd}\hat{y} \cos \varphi_{dr}$, with $C_{pd} = \frac{-C_{pp}}{C_{dd}}$ (9)

If $\omega = \omega_d$ (displacer is tuned to the running frequency) $\varphi_{dr} = 0$, and thus no cold production takes place.
If $\omega_d > \omega$, then $\varphi_{dr} > 0$ and the machine can run as a refrigerator.

In fig. 5 $\hat{x}$ and φ_{dr} are given as a function of ω_d/ω. When ω_d/ω increases, φ_{dr} increases but $\hat{x}$ decreases.
(9) substituted in (6) gives:

$$Q_{eo} = -n\pi C_y C_{pd} S_d \hat{y}^2 \sin \varphi_{dr} \cos \varphi_{dr}$$

The optimum will be at $\varphi_{dr} = 45°$ thus

$$Q_{eo\ max} = \frac{-n\pi C_y S_d C_{pd} \hat{y}^2}{2}$$

Then according to (9)

$$\frac{M_d(\omega_d^2 - \omega^2)}{C_{dd}S_d\,\omega} = 1 \text{ or } C_{vd} = M_d\omega^2 + C_{dd}S_d\omega$$

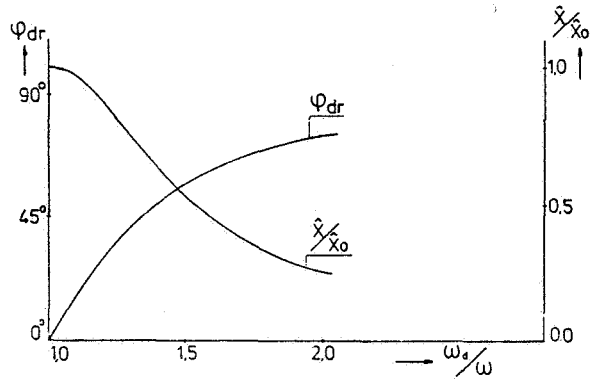

Fig. 5. - Phase shift and relative amplitude of the displacer versus ω_d/ω

Introducing a friction force described as the first harmonic of $F_{fr} = F\,\dot{x}/|\dot{x}|$ (8) then becomes

$$\tan \varphi_{dr} = \frac{M_d(\omega_d^2 - \omega^2)}{C_{dd}S_d\omega + 4F/\pi/\hat{x}}$$

and (9)

$$\hat{x} = C_{pd}\hat{y} \cos \varphi_{dr} - \frac{4F/\pi}{C_{dd}S_d\omega}$$

Both amplitude $\hat{x}$ and φ_{dr} have been decreased slightly so that the maximum of Q_{eo} will decrease and will be at a slightly smaller value of φ_{dr}.

MOVEMENT OF THE PISTON

The equation of movement of the piston is

$$M_p\ddot{y} + (p_c - p_m)S_p + B_g l_v i + C_{vp}y = 0 \qquad (10)$$

in which $M_p\ddot{y}$ is the acceleration force (M_p is mass of moving part), $(p_c - p_m)S_p$ the gas force on the piston, $C_{vp}y$ the spring force and $B_g l_v i$ the electromagnetic force. B_g is the magnetic field in the gap, l_v the length of wire present in the gap and i the current through the coil [5] (the friction force is neglected).

The electric current, neglecting the inductance of the coil, is given by:

$$i = \frac{U_o \cos(\beta + \varphi) - U_m}{R_i} = \frac{U_o \cos(\beta+\varphi) - B_g l_v \dot{y}}{R_i}$$

$U_o \cos(\beta + \varphi)$ is the voltage of the source ($\beta = \alpha - \varphi_{dr}$ and φ is the fase between voltage and piston movement), U_m is the inductive voltage due to the movement of the coil and thus equal to $B_g l_v \dot{y}$.
When substituted in the equation of motion (10), this gives:

$$(M_p\omega^2 - S_pC_y - C_{vp})\hat{y} \cos \beta + B_c\omega\hat{y} \sin \beta = S_pC_x\hat{x} \cos(\beta + \varphi_{dr}) - B_uU_o \cos(\beta + \varphi)$$

The pressure formula has also been used here, and the flow loss has been neglected.
Further

$$y = \hat{y} \cos \beta$$
$$x = \hat{x} \cos(\beta + \varphi_{dr})$$

and $B_c = \frac{(B_g l_v)^2}{R_i}$ and $B_u = \frac{B_g l_v}{R_i}$ have been introduced.

The spring constant for the piston consists of two parts, C_{vp} the mechanical spring constant (which is small) and $C_y S_p$, the spring constant of the gas spring.
Solving the equation (also using eq. (8) and (9) gives:

$$\tan \varphi = \frac{B_c\omega + S_pC_xC_{pd} \sin \varphi_{dr} \cos \varphi_{dr}}{S_pC_xC_{pd} \cos^2 \varphi_{dr} - (M_p\omega^2 - C_yS_p - C_{vp})}$$

Thus φ is independent of the applied voltage (U_o), but strongly dependent on the resonance conditions.

$$\text{Also } \hat{y} = \frac{B_uU_o \sin \varphi}{B_c\omega + S_pC_xC_{pd} \sin \varphi_{dr} \cos \varphi_{dr}}$$

so the amplitude of the piston movement is proportional to the applied voltage.
The power delivered to the piston is:

$$N_p = \frac{1}{T}\int_0^T U_m i\,dt = \frac{\omega\hat{y}}{2}(B_uU_o \sin \varphi - B_c\omega\hat{y})$$

The electric input power is

$$N_{el}=\frac{1}{\pi}\int_0^{\pi} U_o\cos(\beta+\varphi)\,i\,dt=\frac{B_u U_o}{2B_c}(B_u U_o-\omega\hat{y}B_c\sin\varphi)$$

The electric efficiency then is

$$\eta_e=\frac{N_p}{N_{el}}=\frac{A\sin^2\varphi}{(1+A)(A+\cos^2\varphi)}\quad\text{with}\quad A=\frac{N_p}{B_c(\omega\hat{y})^2/2}$$

The maximum efficiency, $\eta_e=\frac{1}{1+A}$, is reached at $\varphi=90°$.
For optimum η_e and optimum Q_{eo}

$$M_p=\frac{C_y S_p+C_{pd}S_p C_x/2}{\omega^2}+C_{vp}\approx\frac{p_m S_p\left(\frac{C_y}{p_m}+\frac{C_{pd}C_x}{2p_m}\right)}{\omega^2}$$

So neglecting C_{vp} the mass of piston plus coil is inversely proportional to ω^2. As M_p cannot be taken too low, for constructional reasons, ω is limited by this condition. Only by increasing $p_m S_p$ to a high value can ω be increased again.
In practice $\omega=314$ (50 cycles) is a very acceptable value.

EXPERIMENTS WITH THE MMC-80

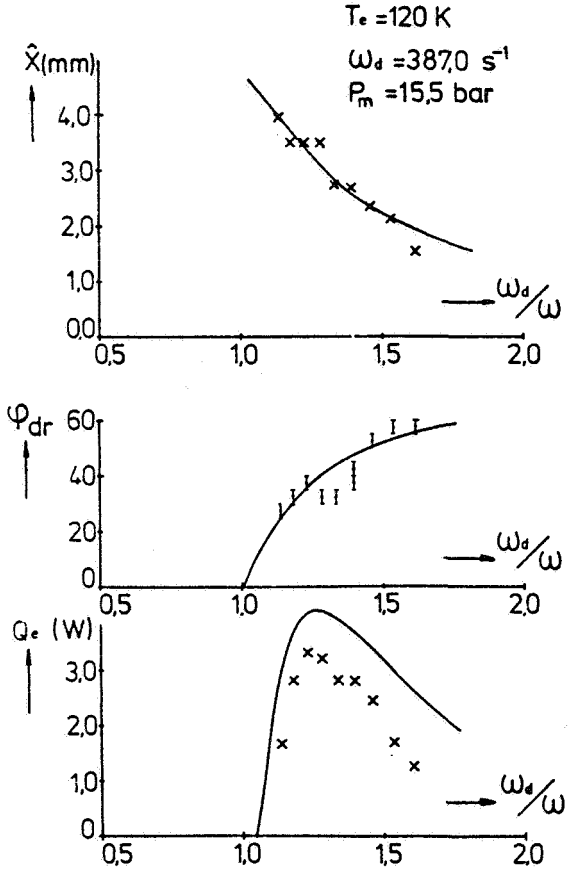

Fig. 6. - Comparison between calculated and experimental results. Displacer amplitude, fase shift and cold production versus ω_d/ω

The experimental results with the MMC-80 have been compared with the above described theory, amended with some effects neglected above. These effects are i.e. the inductance of the coil, the variation of the length of the wire (l_y) present in the gap and the variation of the constants C_x, C_y, C_{pp} and C_{dd} as derived from numerical Stirling calculations.

Fig. fig. 6 the amplitude of the displacer $\hat{x}$, the phase (φ_{dr}) and the cold production Q_e have been plotted against ω_d/ω. In this test the piston amplitude $\hat{y}$, the temperature of the freezer T_e and the average pressure p_m have been kept constant. With an assumed displacer friction of F = 0.5 N, which is a realistic value in this case, the agreement between theory and experiment is reasonably well for $\hat{x}$ and φ_{dr}. The measured cold production is lower than the calculated Q_e, which is most often the case in cold-spot machines where the total heat losses are of the same order as the gross cold production. Thus small errors in the loss calculations lead to large errors in net cold production.

In fig. 7 results are given of measure-

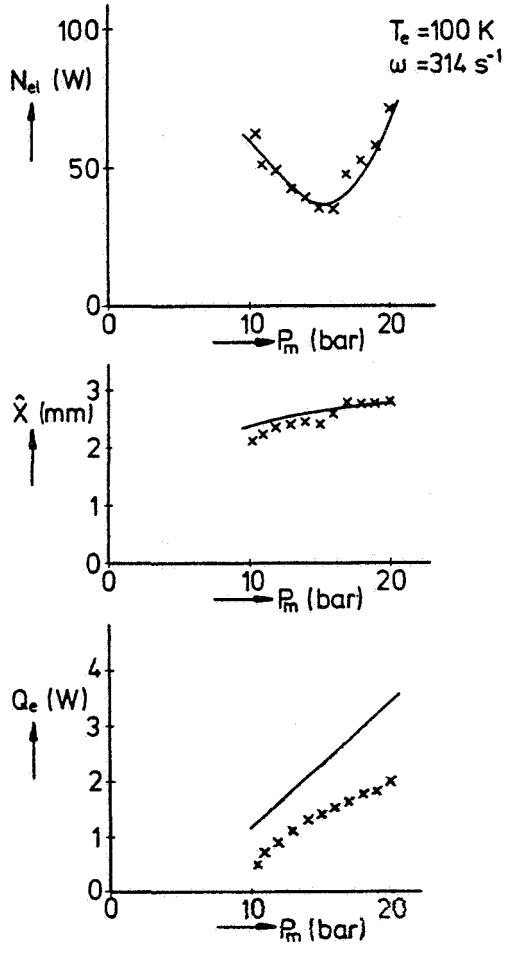

Fig. 7. - Electrical input, displacer amplitude and cold production versus mean pressure with constant ω

ments where p_m has been varied and ω kept constant. In that case the theory predicts a constant φ_{dr} and $\hat{x}$, a linear variation of Q_{eo} and N_p, versus p_m (the piston stroke $\hat{y}$ is kept constant).
As the electric power N_{el} is given by:

$$N_{el} = N_p/\eta_e = B_c(\omega\hat{y})^2/2 \; \frac{(1+A)(A+\cos^2\varphi)}{\sin^2\varphi}$$

N_{el} will show a minimum at a value of p_m where the piston is in resonance.
Here again good agreement has been found between theory and experiment.

CONCLUSION

The above method of describing the pressures inside the Stirling cycle as a vectorial sum of different terms depending only on the respective positions and velocities of both displacer and piston offers a simple way of understanding the characteristics of a free displacer Stirling refrigerator driven by a linear motor.

However, a more detailed numerical calculation of the different constants is needed for design purposes. The variation in the constants, and some effects which have been neglected in the simplified theory are yet so large that they cannot be disregarded if good agreement is to be obtained between theory and experiment.

REFERENCES

1. G.J. Haarhuis, "A Magnetically Driven Stirling Refrigerator". ICEC 7 London, p.422 1978.

2. J.W.L. Köhler and C.O. Jonkers, "Fundamentals of the Gas Refrigerating Machine". Philips Techn. Rev. 16, p.69, 1954.

3. R.J. Meijer, "The Philips Stirling Engine". Ingenieur, vol. 81, May 1969, p. W69, W81.

4. G. Prast and A.K. de Jonge, "A Free Piston Stirling Engine for Small Solar Power Plants". 13th IECEC p.182, 1978.

5. J. Polman, A.K. de Jonge and A. Castelijns, "Free Piston Electrodynamic Gas Compressor". Proc. 1978 Purdue Compressor Technology Conference, Purdue University 1978, p.241.

NOMENCLATURE

p	pressure
T	temperature
τ	temperature ratio
S	surface area
M	mass
V	volume
B	total volume, normalised to T_c and reduced to V_o
x	displacer position
y	piston position
ω	angular frequency
φ	fase shift between piston and applied voltage
φ_{dr}	fase shift between piston and displacer
n	cycles per second
t	time
α	ωt
β	$\alpha - \varphi_{dr}$
f	$1/t$
Q	cold production
N	power
η	efficiency
U_o	amplitude of the applied voltage
U_m	inductive voltage due to the movement of the coil
R_i	resistance of the coil
B_g	magnet field in the gap
l_v	length of wire present in the gap
i	electric current
B_c	constant = $(B_g l_v)^2/R_i$
B_u	constant = $(B_g l_v)/R_i$
A	ratio between N_p and $B_c(\omega\hat{y})^2/2$
F_{fr}	friction force acting on the displacer
C_x	constant of pressure variation due to displacer position
C_y	constant of pressure variation due to piston position
C_p	constant of flow loss due to piston velocity
C_d	constant of flow loss due to displacer velocity
C_{pp}	$= C_{pe} - C_{pc}$
C_{dd}	$= C_{de} - C_{dc}$
C_{pd}	$= -C_{pp}/C_{dd}$
C_v	spring constant

Subscripts

m	mean value
c	compression side
e	expansion side
i	volume i
d	displacer
p	piston
0	reference value